“Satisficing” Viable Solutions to a Monetary Policy Problem

J. B. Krawczyk and R. Sethi
School of Economics & Finance, Victoria University of Wellington
and
Economics Department, Reserve Bank of New Zealand
New Zealand

Abstract

Computing the optimal trajectory over time of key economic variables is a standard exercise in the analysis of many macroeconomic systems. In practice, however, it may be enough to ensure that these variables evolve within certain bounds rather than optimally. In this paper we study the problem of setting monetary policy in a “good enough” or satisficing sense, rather than in the optimising sense more common in the literature. Important advantages of our approach over policy optimisation include greater robustness to model, parameter, and shock uncertainty, and an adequate characterisation of otherwise imprecisely defined monetary policy goals.

Our analysis frames the monetary policy problem in the context of mathematical viability theory, which rigorously captures the notion of satisficing. A solution to a viability problem is a set of initial conditions for the system, for which there exist strategies that maintain the system within a desired state domain.

We estimate a simple small open economy model on New Zealand data and use viability theory to discuss the possibility of maintaining inflation, output, and interest rate within some acceptable bounds. We derive interest rate rules that achieve such an outcome endogenously.

1 Introduction

The aim of this paper is to explore usefulness of viability theory for an analysis and synthesis of a monetary policy problem.\(^1\) We use a stylised monetary policy control problem – that faced by the Reserve Bank of New Zealand – as a vehicle to demonstrate the nature of a solution to a viability problem.\(^2\)

Herbert A. Simon, the 1978 Economics Nobel Prize laureate, talked about satisficing (his neologism) rather than optimising, as being what the economists really need. We

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\(^1\)This paper draws from companion papers [15] and [17] and their more recent working paper versions.

\(^2\)The views expressed are those of the authors and are not necessarily endorsed by the Reserve Bank of New Zealand.
think that economic theory, which follows the Simon prescription, brings modelling closer to how people actually behave. We also think that it is *viability theory*, which is a relatively young area of continuous mathematics (see [2] and [3]), that rigorously captures the essence of *satisficing*. Therefore, viability theory appears to be an appropriate tool of achieving a *satisficing* solution to many economic problems. We aim to demonstrate this by solving a stylised Reserve Bank of New Zealand macroeconomic stabilisation problem. The solution will enable us to analyse the system’s *evolution* within certain normative constraints (such as a desired inflation band) rather than the system’s *convergence* to a steady state, as is the case with many traditional (optimal) monetary policy solutions. On the basis of the system’s evolution we will be able to propose some robust *satisficing* monetary policies.\(^3\)

Policies obtained through viability analysis are “robust” or precautionary or preventative in that they are based on the economic system’s *inertia*. This makes them naturally forward looking and suitable for “any” future circumstances. This is so because knowledge of the system’s inertia enables detection and avoidance of regions of economic certain conditions (such as large output gap or accelerating inflation) where control of the system is difficult or impossible.

In this paper, we estimate a simple macroeconomic policy model using historical New Zealand data. We show how the central bank may deal with the inflation stabilisation problem.\(^4\)

In the next section, we provide a brief introduction to viability theory and, in Section 3, we apply it to a simple macroeconomic model. The paper ends with concluding remarks.

2 What is viability theory?

2.1 Definitions

Viability theory is an area of mathematics concerned with *viable evolution* of controlled dynamic systems. In broad terms, a dynamic system evolves “viably” if the system state variables remain in a prescribed region of the state space for as long as the evolution is concerned.

Suppose a system evolution is expected to satisfy some normative constraints, which define a closed set of the phase space. If there exists a system trajectory that remains in this set for as long as the evolution is concerned, such evolution and the set are called *viable*. The basic problem that viability theory attempts to solve is whether a control strategy exists that prevents the system from leaving the constraint set. The *viability kernel* (formally defined in Definition 2.1), for such a problem is the set of all initial conditions, for which such a strategy exists.

\(^3\)Actually, *alternative* robust strategies. For min-max strategies applied to the design of robust monetary policies, see e.g., [13], [30] and [29]. Strategies delivered by viability theory are *alternative* because they are not based on min-max optimisation but on an evolutionary analysis of admissible system trajectories.

\(^4\)Notwithstanding this macroeconomic application, viability theory can be applied to other problems where uniqueness of the optimal strategy is not of major concern. See [26] and [8] where an endogenous business cycle was studied and [7] where an analysis of exchange rate dynamics was carried out. For viability theory applications to environmental economics see [4] and [20]; see [24] and references provided there for applications to financial analysis.
Consider a dynamic economic system with several state variables. At time $t \in \Theta \equiv [0, T] \subset \mathbb{R}^+$, where $T$ can be finite or infinite, the state variables are

$$x(t) \equiv [x_1(t), x_2(t), \ldots, x_N(t)]' \in \mathbb{R}^N, \quad \forall t \in \Theta$$

and the controls (or actions) are

$$u(t) \equiv [u_1(t), u_2(t), \ldots, u_M(t)]' \in \mathbb{R}^M, \quad \forall t \in \Theta.$$  

Imposition of normative restrictions on states and strategies means that

$$\forall t \in \Theta, \quad x(t) \in K \quad \text{and} \quad u(t) \in U$$

where symbols $K$, $U$ represent compact sets of constraints that the state and control variables need to satisfy. In general, the control constraints depend on $x$; however, for simplicity, we will avoid notation $U(x)$.

The state evolves according to system dynamics $f(\cdot, \cdot)$ and controls $u(t)$ as follows

$$\dot{x}(t) = f(x(t), u(t)) \quad t \in \Theta, \quad x(t) \in K \subset \mathbb{R}^N, \quad u(t) \in U \subset \mathbb{R}^M. \quad (1)$$

Evidently, at every state $x(t)$, the system’s velocity $\dot{x}(t)$ depends on action $u(t)$. We may say that the velocity at $x(t) \in K$, for any $t \in \Theta$, is governed by the set-valued map (or correspondence)

$$F(x) \equiv \{f(x, u), u \in U\}. \quad (2)$$

Combining the above formulae, the system’s dynamics can be rewritten in form of a differential inclusion:

$$\dot{x}(t) \in F(x(t)), \quad \text{for almost all} \quad t \in \Theta, \quad (3)$$

which determines the range of velocities of the state variables at $x(t)$.

In economic terms, the last relationship tells us that at time $t$, for a given composition of $x$ (capital, labour, technology, etc.), the extent of growth (or decline), or steady state stability, are all dependent on the map $F : \mathbb{R}^N \to \mathbb{R}^N$ whose values are limited by the scope of the system’s dynamics $f$ and controls contained in $U$.

A viability theory analysis attempts to establish non-emptiness of a viability kernel, which is a collection of loci for initial conditions of viable evolutions $x(t), t \in \Theta$.

**Definition 2.1.** The **$T$-viability kernel** of the constraint set $K$ for the control set $U$ is the set of initial conditions $x_0 \in K$ denoted as $V^K_F(T)$ and defined as follows:

$$V^K_F(T) \equiv \{x_0 \in K : \exists x(t) \text{ solution to (3) with } x(0) = x_0 \text{ s.t. } x(t) \in K, \forall t \in \Theta\}. \quad (4)$$

In other words, we know that if a trajectory begins inside the viability kernel $V^K_F(T)$ then we have sufficient controls to keep this trajectory in the constraint set $K$ for $t \in [0, T]$ (we have denoted the latter set $\Theta$). See Figure 1 for an illustration of the viability idea when $T = \infty$ (autonomous systems).

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5 The state may be generalised to meta-state, which will comprise instruments (flows) along the usual stock variables. If so, the system’s controls will be velocities of the instruments; see later in the paper the system of equations (23)-(25), page 15.

6 The control (or action) set $U$ could be split into two (or more) subsets if there were two (or more) players who would decide upon the actions.

7 If $V^K_F(T) = \emptyset$ then we can say that $K$ is a repeller.
The state constraint set $K$ is represented by the yellow (or light shadowed) round shape contained in the state space (here, $X \equiv \mathbb{R}^2$). The solid and dash-dotted lines symbolise system evolution.

The viability kernel for the constraint set $K$, given controls from set $U$ and the system dynamics’ correspondence $F$, is the purple (darker) shadowed contour denoted $V^K_F(T)$. The system evolution represented by the trajectories that start inside the kernel (dashed lines) are viable in $K$ i.e., they remain in $K$. This is not the property of the other trajectories (dash-dotted lines) that start outside the kernel. They leave $K$ in finite time $< T$. If policy instruments were allowed to depend also on time i.e., if they were $u(x, t)$ rather than just $u(x)$, then solid lines could leave $V^K_F(T)$ but would remain in $K$.

We can now say what we understand as a viability problem, and define what we mean by its solution.

Given the system correspondence $F(\cdot)$ and sets of constraints $K$ and $U$, the associated viability problem consists of establishing existence of the viability kernel $V^K_F(T)$.

**Definition 2.2.** When the kernel is nonempty $V^K_F(T) \neq \emptyset$, we say that the viability problem has a solution; otherwise, the viability problem has no solution.

Let us observe that many real-life economic-dynamics’ problems are non-stationary. For example, any finite horizon problem or one that depends on a season is non-stationary. In general, methods and the amount of effort for optimal solutions differ between non-stationary and stationary problems. However, viability theory allows us to treat finite and infinite horizon problems uniformly.

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8Notice that traditional monetary-policy optimisation models (e.g., [27]) are modelled and solved as infinite horizon and stationary.
2.2 Geometric characterisation

Viability kernels can be characterised relatively easy for some typical dynamic systems (see [2], [3], [6]).

Consider the linear dynamics system defined as follows:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} =
\begin{bmatrix}
1 & -1 \\
1 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} +
\begin{bmatrix}
v_x \\
v_y
\end{bmatrix}.
\]  

(5)

The state variables are \(x, y\); the instrument set \(U\) is the unit ball

\[U \equiv \{(v_x, v_y) : v_x^2 + v_y^2 \leq 1, \quad (v_x, v_y) \in \mathbb{R}^2\}\]  

(6)

where \(v_x\) adds to speed in direction \(x\) and \(v_y\) adds to speed in direction \(y\).

We can see that the further from origin the system is the faster it moves along direction \(y\). This might remind us of a paddling boat on a river approaching a waterfall. The further we are from calm waters \((y = 0)\) the faster we move. Intuitively, given our limited strength, we sense existence of the point of no return, after which we will not be able to paddle away from the waterfall.

Notice that this situation is reminiscent of what might occur if an economy is hot. The larger the output gap, the higher inflation, and so a larger still output gap results, etc.. Given limited instruments at the Central Bank disposal, too hot an economy might be bound to suffer from a spiraling inflation. Clearly, determination of the point of no return on the river and of the combination of output gap and inflation for the economy is important for a safe journey. In viability theory, these points are determined once the viability kernel is established.

First, we will check if rectangle

\[Q \equiv \{(x, y) : \max(|x|, |y|) \leq 1, \quad (x, y) \in \mathbb{R}^2\}\]

is a viability kernel. If it is, we will be certain that we can prevent the system from escaping from \(Q\) using actions from \(U\).

Figure 2 provides an illustration of the problem and of some geometric properties required for viability.

Consider the frontier point \(B\) of the rectangle. The velocities from \(U\) are constrained (see (6)) and generate evolution directions denoted by “x“ (crosses); the normal\(^9\) at \(B\) is the thin black line. We see that all angles are acute and that there is no vector at \(B\), which would point inside the rectangle (or, form an obtuse angle with the normal). We see that for any \((v_x, v_y) \in U\), we are unable to “return” to the rectangle from point \(B\). This means that point \(B\) is not viable. The same reasoning can be repeated at many points of rectangle \(Q\). The conclusion will be that the rectangle cannot be the viability kernel.

However, we can prove that disc \(Q_B\)

\[Q_B \equiv \{(x, y) : x^2 + y^2 \leq 1, \quad (x, y) \in \mathbb{R}^2\}\]

\(^9\)Actually, proximal normal. Given a convex set and a point outside the set, in a normed space, a proximal normal is the direction of a vector that connects the point to a nearest point of the set. A normal is just a direction on which a support functional is positive. We assume our problems are formulated in normed spaces and will use the term normal.
(delimited by the circle of radius 1 centred at origin) is a viability kernel. Indeed, we see that there are velocities at point \( A \), which generate evolution directions represented by "*" (stars), that point inside the disc (they form an obtuse angle with the normal going through this point). This means that there exists some \((v_x, v_y) \in U\) for which we can "turn" the system so that it remains in \( Q_B \). This reasoning can be repeated at any point of disc \( Q_B \). We could also prove that no point \((x, y) \in Q \setminus Q_B\) is viable and conclude that disc \( Q_B \) is the viability kernel.

The evolution direction at \( A \) that keeps the system inside \( Q_B \) is represented by the black vector at \( A \) pointing left. This direction is a consequence of use of some "outermost" velocities \((v_x, v_y) \in U\). Presumably, this control is extreme (or "outermost") in that there is no other velocity vector, which would generate a more obtuse angle with the normal at \( A \). In fact, the pair of velocities satisfies that generate the black vector satisfies \( v_x^2 + v_y^2 = 1 \).

A comparison between sets \( Q \) and \( Q_B \) (the latter is a viability kernel the former is not) tells us that at \( B \), the system moves too "fast" to be controlled through \((v_x, v_y) \in U\). However, the same control set contains elements that are sufficient to restrain the system, should we apply them "early"\(^{10}\) i.e., when the process is within \( Q_B \).

### 2.3 Viable policies

In economic situations, in which a "planner" may be identified (e.g., central bank), the establishment of a viability kernel can be used to select policies that keep the dynamic process \( x \) inside the closed constraint set \( K \). Once the kernel is established, choosing a satisficing policy is a simple procedure, which can be illustrated using Figure 2. Before

\(^{10}\)Suppose the evolution started at origin.
we explain the procedure let us briefly look at what kind of actions a central bank planner undertakes.

Routinely, every given time interval, the planner announces a cash interest rate. A Taylor rule or an optimising rule\(^{11}\) might be used to determine the “new” interest rate. The latter usually equals the old interest rate plus or minus a fraction of a percentage point. The process leading to the rate determination is typically based on implicit or explicit optimisation of a loss function, which contains a significant number of parameters calibrated and/or estimated.

Looking back at Figure 2 we can see that there are controls \(v_x^2 + v_y^2 \leq 1\) that keep trajectory \(x\) in \(Q_B \subseteq Q\). In particular, we know that even if \(x(t)\) is at the frontier of \(Q_B\) the “outermost” or extreme control is sufficient to prevent the system trajectory from leaving \(Q_B\).

We will term extreme a control from set \(U\) that belongs to this set’s boundary. We will make the meaning of this definition clear every time a viability kernel will be determined.\(^{12}\)

Denote \(\text{fr}D\) the frontier of the viability kernel \(D\). For deterministic models, the following satisfying policy prescription follows from the above:

\[
\begin{align*}
\{ & \quad \text{if } x(t) \in D \setminus \text{fr}D, \quad \text{apply any control } \in U; \\
& \quad \text{if } x(t) \in \text{fr}D, \quad \text{apply extreme control } \in U
\end{align*}
\]

(7)

The effect of the Central Bank optimisation process is similar to the application of the satisfying policy: either maintains \(x\) (e.g., inflation) in \(K\). However, as will be explained later, fewer (subjective) parameters are needed to establish \(D\) than to compute a minimising solution to the bank loss function. Also, the “relaxed” approach advocated by (7) (the first “if”) offers the planner the possibility to strive to achieve other goals (e.g., political), which were not used for the specification of \(K\). (Perhaps they were difficult to specify mathematically or they arose after the viability kernel had been established.) This is not the case of an optimal solution that remains optimal for the original problem formulation only.

Should there be uncertainties regarding the model parameters, a sensitivity analysis needs to be performed to establish the extent to which the system dynamics are affected by the uncertainties. Once established, the current position of the system will be generalised from \(x(t)\) to \(x(t) + b_1(x(t), \kappa(t))\) where \(b_1(\cdot, \cdot)\) is a ball centred at \(x(t)\) with radius \(\kappa(t)\) that will result from a robustness analysis of (3).

When the model is subjected to shocks whose magnitude can be estimated (or their distribution is known), the viability kernel will have to be such that \(x(t) + b_2(x(t), \varepsilon(t)) \in K\) where the radius \(\varepsilon(t)\) will depend on the shock\(^{13}\). Then, the above policy prescription can be followed.

\(^{11}\)See e.g., [27].

\(^{12}\)Element of \(U\) that guarantees viability of \(Q_B\) in Figure 2 is extreme because \(v_x^2 + v_y^2 = 1\).

\(^{13}\)This radius might equal a standard deviation shock magnitude. It may also equal the size of the shock that occurs “once in 100 years”. Etc.
3 A macroeconomic model

3.1 A viability theory problem

It is not unreasonable to characterise a typical central bank as one that seeks to maintain a select few macroeconomic variables within some bounds. This claim finds support in legislation, in government-delegated mandates, and in loss-functions for central banks commonly employed for monetary analysis. These variables ordinarily include inflation, output, exchange and interest rates, and precise goals for these variables can be specified in both levels and in growth rates.

As noted above, the bank realises its multiple targets using optimising solutions that result from minimisation of the bank’s loss function. Typically, the loss function includes penalties for violating an allowable inflation band and also for non-smooth interest adjustment. The solution, which minimises the loss function, is unique for a given parametrisation of the bank’s loss function. As such, alternative strategies are not allowed for.

We now turn to the main goal of this paper: to establish a viability kernel for a stylised monetary policy problem as faced by the Reserve Bank of New Zealand. We establish the economic kernel, the subset of the constraint set $K$, inside which the evolution of the economy can be contained, given reduced-form economic dynamics and instruments available to the central bank.

In the next section we will describe a stylised monetary rules model (inspired by [27] and [28])) and estimate the parameters using historical New Zealand data on inflation, output gap and nominal interest rate. We will then show how viable, and statisticising, policies can be obtained for that model. We will also highlight how the solutions obtained through viability theory do not suffer from drawbacks typical of their optimising counterparts.

3.2 A Central Bank problem

The Policy Targets Agreement negotiated between the New Zealand Government and the Reserve Bank of New Zealand currently requires the latter to maintain inflation between 1 and 3 per cent on average over the medium-term. Maintenance of this goal is subject to the further requirements that unnecessary volatility in output, interest and exchange rates be avoided. In the model below, we assume that the central bank uses a short-term nominal interest rate $i(t)$ as a policy instrument to control inflation $\pi(t)$ and, to a lesser extent, output gap $y(t)$. Given our focus on the application of viability theory we use a popular, but simple, new-Keynesian model. See [27] for a derivation of this model from microeconomic constructs and an examination of its properties.

The model is given by the equations

\begin{align}
    y(t) &= a_1 y(t - h) + a_2 y(t - 2h) - a_3 \left( i(t - h) - E_{t-h} \pi(t) \right) + u(t) \quad (8) \\
    \pi(t) &= \pi(t - h) + \gamma y(t) + \eta(t) \quad (9)
\end{align}

where $y(t)$ is the output gap, $\pi(t)$ is inflation, $i(t)$ is a nominal interest rate and $u(t), \eta(t)$ are serially uncorrelated disturbances with means equal to zero. The parameters $a_1, a_2, a_3, \gamma$ are estimated.
Equation (8) is an aggregate demand relationship. It corresponds to a traditional IS function where aggregate demand is inversely related to the real interest rate \( r(t - h) = i(t - h) - E_{t-h}\pi(t) \). Note that, in our aggregate spending specification, time-\( t \) spending depends on the lagged value of the real interest rate. Since monetary policy affects aggregate demand via the real interest rate, the assumption that time-\( t \) spending depends on the lagged real interest rate will imply a lagged response of output to monetary policy changes. This reflects a long-standing view that many macroeconomic variables do not respond instantaneously to monetary policy shocks. The interest rate relevant for aggregate spending decision is the long-term rate, which is related to the short-term rate via the term structure relationship. To minimise the number of variables in our exposition, we do not distinguish between the long-term and short-term interest rates.

Equation (9) captures the inflation-adjustment process driven by the size of the output gap. In the canonical new-Keynesian specification, current inflation \( \pi(t) \) depends on expected future inflation \( E_t\pi(t+h) \). Furthermore, in the Fuher-Moore ([12]) model of multi-period, overlapping nominal contracts, current inflation depends on both past inflation and expected future inflation. However, several empirical works (e.g., [11]) suggested that the expected inflation term is empirically unimportant once lagged inflation is included in the inflation adjustment equation. Considering this, and also to simplify our exposition, we ignore the expected inflation term. Given that the output gap enters contemporaneously in the Phillips curve, it is necessary to ensure that our system is identified. We verify that both the order and rank conditions for identification is satisfied.

Hence, our model is a simplified version of the Rudebusch-Svensson, see [25], with output gap and inflation as state variables.

Assume \( a_2 = 0 \) in (8), call \( a \) the “new” coefficient with the one-lag term and apply the expectation operator \( E_{t-h} \) to both (8), (9). We obtain

\[
E_{t-h}y(t) = a E_{t-h}y(t-h) - a_3 \left( E_{t-h}i(t-h) - E_{t-h}\pi(t) \right) \tag{10}
\]

\[
E_{t-h}\pi(t) = E_{t-h}\pi(t-h) + \gamma E_{t-h}y(t) \tag{11}
\]

At time \( t-h \), expectations are identical with observations so,

\[
E_{t-h}y(t) = a y(t-h) - a_3 \left( i(t-h) - E_{t-h}\pi(t) \right) \tag{12}
\]

\[
E_{t-h}\pi(t) = \pi(t-h) + \gamma E_{t-h}y(t). \tag{13}
\]

Assume differentiability of the inflation and output gap processes. If so, for small \( h \)

\[
E_{t-h}y(t) = y(t-h) + \dot{y}h \tag{14}
\]

\[
E_{t-h}\pi(t) = \pi(t-h) + \dot{\pi}h. \tag{15}
\]

These relationships tell us that agents forecast the expected values by extrapolating from past observations. This corresponds to the basic learning process (compare [14]).

\[\text{We have imposed the accelerationist property on the Phillips curve in that the lag of inflation is restricted to have a co-efficient of one. The implication, therefore, is of a Phillips curve that is vertical in the long run.}\]
Substituting in (12), (13) (and omitting the time index \( t - h \)) yields:

\[
\begin{align*}
y + \dot{y}h &= a y - a_3 \left( i - (\pi + \dot{\pi}h) \right) \\
\pi + \dot{\pi}h &= \pi + \gamma(y + \dot{y}h).
\end{align*}
\]

(16) \hspace{1cm} (17)

From (17), \( \dot{\pi}h = \gamma y + \dot{y}h \). Allowing for that and for \( \alpha h = a - 1 \), \( \xi h = a_3 \), \( \zeta h = \gamma \), then dividing by \( h \) and requesting \( h \to 0 \) we get the following inflation and output gap dynamics

\[
\begin{align*}
\frac{dy}{dt} &= \alpha y(t) - \xi \left( i(t) - \pi(t) \right) \\
\frac{d\pi}{dt} &= \zeta y(t).
\end{align*}
\]

(18) \hspace{1cm} (19)

The equations, which constitute the above model, are continuous time equivalents of the aggregate demand equation (8) and the Phillips curve (9). They state that the output gap constitutes a “sticky” process (18) driven by the real interest rate \( i.e. \), the difference between the interest and inflation rates and that inflation (19) changes proportionally with the output gap. After we estimate the model in Section 4 we will show a few possible time profiles of the variables of interest and how the model can be used to determine an appropriate interest rate response to inflation and demand shocks.

The model given by (18)-(19) provides insight into the formation of expectations of inflation and the output gap by private sector agents. For example, positive and increasing output gaps imply that an increase in \( i(t) \) above its neutral rate will eventually cause the output gap to close. Further, inflation will will not begin to decrease as long as output gap remains positive.\(^{15}\) We observe that the central bank may exploit the inertia of controlled processes to constrain their evolution.

4 Estimating the model

There are severe limitations in taking the above model to the New Zealand data. The equations describe a closed economy, with no role for exchange rates or other external economy influences. However, we persist with this model for two reasons. First, this note presents an illustrative application of viability theory, and as such, we are keen to reduce to the dimensionality of the problem to its bare minimum to allow for clearer exposition. Second, we believe that even as a very basic representation of the New Zealand economy, viability analysis applied to the model above yields lessons that are informative for central bank decision-makers. In the next step of our research we intend to extend the model by including an open-economy element in the exchange rate. (See [18].)

4.1 New Zealand historical data

There are several options available in moving from the theoretical model to its empirical counterpart. We initially considered defining \( \pi_t \) as non-traded inflation on the basis of the closed nature of the model, and given that the traded and non-traded components of

\(^{15}\)In this simple model, inflation grows for any positive output gap. However, adding an exchange rate to the model as in [18] helps to understand why this might not always be the case.
inflation exhibit very different dynamics, the use of non-traded inflation in estimation is likely to yield a better fit since it abstracts from exchange rate effects. Second, and again in the interests of arriving at a better fit from a parsimonious model, there is a well-documented link between non-traded inflation and the Reserve Bank’s multivariate output gap (see [5]). This relationship is a cornerstone of the Bank’s main macro-model for forecasting and policy analysis. However, this relationship does not hold for the aggregate CPI. One important disadvantage of using non-traded inflation rather than aggregate CPI is that the former does not map directly to the Bank’s inflation target.

![Figure 3: Estimation data](image)

We prefer to use the output gap resulting from the MV filter rather than a more standard HP-filtered. The MV-gap incorporates a HP-type filter together with an unemployment gap and the level of capacity utilisation.

For inflation expectations we have the choice of using either data from the Reserve Bank’s Survey of Expectations and assuming some learning about the policy formation process, or alternatively the choice of assuming rational expectations. There also remain the possibilities of assumptions of perfect foresight or that the expectations formation process is entirely backward looking. It is important to note that surveyed expectations measure CPI inflation rather than non-traded inflation. Given our desire to keep the model dynamics simple, the assumption of rational expectations is unsuitable since it adds an extra state variable to the analysis.

In light of the above discussion we use the following data definitions: $\pi(t)$ is CPI inflation, $y(t)$ is a multivariate (MV) output gap, and $i(t)$ is the nominal 90-day interest rate.
rate which is a good proxy for the actual policy interest rate, the Official Cash Rate. Finally, we use one-quarter ahead inflation expectations from the Reserve Bank’s survey.\footnote{We do not observe any significant difference in the estimation results from using backward looking expectations or perfect foresight. Additional estimation details are available on request.} Figure 3 charts the series over the sample period 1992:1 to 2005:4.

Before we turn to a discussion of the estimation results it is important to note that the model is framed in gap terms and that our de-trending procedure to render the model stationary is a simple demean of the data. However, in later analysis of the viability kernel we prefer to use these variables in levels since they relate more closely to those that the policymaker actually has preferences over. Consequently, in later analysis we add the mean back to the variables. In this context, the difference of the nominal interest rate and the inflation target in levels terms can be interpreted as a proxy for the neutral real interest rate.

4.2 Estimation results

With the empirical mapping of the theoretical model outlined, we now proceed to estimate the model using single equation least-squares methods. These results are presented in Table I and the fit of the data to the two model equations (together with estimated residuals) is shown in Figure 4.

<table>
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<th>IS summary statistics</th>
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</tr>
<tr>
<td>$\gamma$</td>
<td>0.047</td>
<td>1.43</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table I: Estimated parameters

The parameter estimates for the Phillips curve are lower than previously estimated on New Zealand data. For example, \cite{19} finds that the coefficient on the output gap in the Phillips curve is 0.15 when estimating a Phillips curve with similar long run properties to ours.\footnote{More specifically, \cite{19} uses GMM and estimates a coefficient of 0.077 on both the contemporaneous and the first lag of output, i.e. yielding a total coefficient 0.152 on the output gap.} The low explanatory power of the curve is likely to be due to the closed economy nature of the model.

The estimated aggregate demand equation delivers much better fit and is broadly plausible. Other Reserve Bank studies report similar estimates.\footnote{See \cite{5} and \cite{19}.}
4.3 The residuals

Turning to the estimated residuals it appears that those of both curves are mildly biased upward. The estimated size of the residuals standard error – 0.5 percent for the IS curve and 0.15 percent for the Phillips curve – matches well with the empirical standard deviations of the output gap and inflation, and thus provides some assurance that a model is a reasonable first approximation to the New Zealand economy.

Consistent with the fact that inflation has been more volatile in the second-half of our sample period, the Phillips curve residuals too exhibit greater variation over this period. Given the relative absence of clear outliers in the residuals, we do not believe that any one observation is overly distorting the fit of the model. Tests of first-order autocorrelation in the residuals for both curves are rejected at the five per cent level.
5 Model completion

5.1 The constraints

Usually there is little doubt what the politically desired inflation bounds are. In New Zealand, the inflation band has been legislated to be confined to \([-0.01, 0.03]\). More specifically, the Reserve Bank is required to keep inflation in this interval on average and over the medium term. In this analysis, however, we impose the more convenient (and more stringent) requirement on the evolution of the model economy in every period rather than in a multi-period average sense.

There is less agreement about what the desired output gap should be. We will assume a rather wide interval for output gap \(y(t) \in [-0.04, 0.04]\) to reflect a secondary concern of the central bank for this variable.

As with the desired size of the output gap, the instrument set composition also depends on political decisions. We assume\(^1^9\) that \(i(t) \in [0, 0.077]\).

An \(\mathbb{R}^3\) region

\[
\mathbb{R}^3 \supset K \equiv \{(y(t), \pi(t), i(t)) : -0.04 \leq y(t) \leq 0.04, 0.01 \leq \pi(t) \leq 0.03, 0 \leq i(t) \leq 0.077\}, \quad (20)
\]

within which the meta-system\(^2^0\) trajectory \([y(t), \pi(t), i(t)]\) will have to be contained, is shown in Figure 5 upper panel and its two dimensional projection in bottom panel.

Together with preferences over inflation and the output gap, central banks also typically have concerns about the degree of interest rate smoothness (see e.g., [1]). That concern is usually modelled by adding \(w\left(i(t) - i(t - h)\right)^2, w > 0\) to the loss function. In continuous time, limiting interest rate changes (or “velocity”)

\[
u \equiv \frac{di}{dt} \quad (21)
\]

produces a smooth time profile of \(i(t)\). Bearing in mind that the central bank’s announcements concern the changes in interest rates, we treat \(u\) as the bank’s control variable. At the Reserve Bank, changes in the policy instrument are usually made every six weeks and a typical change, if made, is 0.25%. The bank’s control set \(U\) can thus be defined as

\[
U \equiv \{u : u(t) \in [-0.005, 0.005]\} \quad (22)
\]

i.e., the interest rate can move by a maximum of 0.5% in every quarter.

Hence, the dynamic system to analyse the relationship between the interest rate, inflation and output gap needs to be augmented by the interest rate velocity constraint. Allowing for the estimation results of Section 4.2 and the relations between the discrete-time and continuous-time model parameters (see Section 3.2, page 10)

\[
\alpha = -0.1103, \quad \xi = 0.1208, \quad \zeta = 0.047
\]

\(^1^9\)Nominal 90-day interest rates have touched 10 per cent within our sample period – June 1996; and the average of the series is close to 6.75% with a standard deviation of 1.5%. We have decided to use an “intermediate” value of 7.7%.

\(^2^0\)See footnote 5, page 3.
we get the following inflation and output gap dynamics:

\[
\frac{dy}{dt} = -0.1103 y(t) - 0.1208 (i(t) - \pi(t)) \\
\frac{d\pi}{dt} = 0.047 y(t). \\
\frac{di}{dt} = u \in [-0.005, 0.005].
\]

5.2 Robustness of model

As we have mentioned in the Introduction, a viability model of the central bank problem needs less subjectively assessed parameters than the corresponding optimisation model. In particular, a viability model (23)-(25)\((plus (22))\) does not require any explicit specification of the weights on a central bank’s loss function. Further, a discount rate is not needed either. The bounds of the constraint set are either legislated or identifiable in a reasonably non-controversial manner. If there is not much concern for limits of a variable, like for output gap, then they can be set to be arbitrarily “large”.

Consequently, the boundaries of the viability kernel, within which the economy can
move, convey information about the desired evolution of the economy in a more objective fashion than the loss function weights and discount factor tell us about agent’s preferences.

6 Viable solutions for deterministic economy

In this section we will assume that the economy described by model (23)-(25) is deterministic. Later in Section 7 we will consider an economy with the same system dynamics, this time subjected to shocks that are distributed as identified in Section 4.3. In each case, results will be the parameter specific; however, the procedure can be easily repeated for any plausible parameter selection.

6.1 A steady state and transition analysis

First, let us examine existence of steady states of (23), (24), (25) and assess their stability. Add the plane $y = 0$ and another one $i = \pi$ to Figure 5, see Figure 6 top panel. The steady states are at the intersection of those planes, see the dash-dotted line in Figure 6 top panel. The bottom panel shows the directions of the system’s evolution on the $y, \pi$ plane.

Figure 6: Constraint set $K$ and steady states.
At any point where the output gap is negative \((y < 0)\) inflation decreases (arrows point south toward “A”); reciprocally, inflation increases if output gap is positive (arrows point north toward “C”). Above the plane \(i = \pi\), interest rate dominates inflation; this causes the output gap to decrease (arrows point left above \(i = \pi\)). Below \(i = \pi\) where inflation is higher than the interest rate the arrows point right, which means that the output gap increases. In sum, for moderate (non-accelerationary) values of \(y\), the output gap diminishes where real interest rate is positive.

Two interesting monetary control problems can be analysed with the help of Figure 6. These problems occur in the three dimensional space (see upper panel) but can be examined more usefully in two dimensions (see bottom panel). First, when an economy is “hot” (i.e., when the output gap is positive and inflation is near its upper boundary), the interest rate \(i\) must increase “early enough” to keep \(y, \pi, i\) in \(K\), else the upper bound on inflation is likely to be violated.

Second, consider what can happen if the output gap is negative and if inflation is close to the lower limit and so there is not much prospect of the output gap closing of its own accord. If the bank lowers the interest rate “too late”, a positive real interest rate will cause a further decrease in the output gap and inflation can decrease to zero where no instrument exists to lift output. If so, the economy may experience a liquidity trap i.e., it will remain in an area where output gap is negative and inflation is close to zero (positive or negative).²¹

Situations like these require the determination of a collection of points from where the control from \(U\) (defined in (22)) is sufficient to keep the economy inside \(K\). Such a collection is the viability kernel defined in Definition 2.1. In the next section we will determine the kernel where the correspondence \(F\) is defined through (23)-(25).

### 6.2 The viability kernel

We now describe the major features of the viability kernel for the two important economic situations that were sketched in the previous section.

#### 6.2.1 An overheating economy

We determine the kernel’s boundaries for the “north-east” corner (C). We restrict our analysis to situations in which the real interest rate in positive. I.e., the economy is above the 45º line in Figure 6 (top panel) and the output gap is decreasing. This monotonic movement enables us to analyse the economy in the two-dimensional space (output gap, inflation) rather in the “full” space that includes also inflation. This will not (necessarily) be the case with analysis conducted in the south-west corner (A).

What should the Reserve Bank target in such an overheating economy? Obviously, any point from which the northern boundary of \(K\) will be exceeded will not be aimed for. This excludes any point on this boundary as a target for as long as \(y(t) > 0\) (see equation (24)). Hence, the Bank will aim at \(y(t) \leq 0\). However, strictly negative output gap \(y(t) < 0\) might mean an impending recession, thus the Bank will pursue a policy to

---

²¹Again, we cite [22] for an analysis of a liquidity trap problem performed through an established method. Also see that [23] is a recent publication where a liquidity trap problem is analysed in state space.
reach $y(t) = 0$. As the the upper bound on inflation in our example is 3% the Bank’s target should be $y(T) = 0, \pi(T) = .03$ for some $T > 0$.

Presumably, as inflation rises toward its upper limit, the nominal interest rate will also rise to its upper boundary. Keeping the latter high may control the inflation but if the real interest rate $i(t) - \pi(t)$ is positive and rather large, a persistent negative output gap can emerge with severe implications for the economy (see equation (23)). Hence, the bank will aim to achieve $y(T) = 0, \pi(T) = .03$ with the interest rate such that minimises pressures on inflation and growth. This means that the Bank will intend $i(T) = .03$. This will be a steady state at which all pressures vanish.

It is therefore of great importance for the Bank to recognise which is the limiting system evolution that reaches $y(T) = 0, \pi(T) = .03, i(T) = .03$ in finite time $T$ and with the maximum allowable interest rate slowdown $u = \frac{di}{dt} = -.005$. The limiting property of the system trajectory for this evolution is such that from all points above this trajectory, the upper limit on inflation will be violated in finite time. Conversely, the steady state $y(T) = 0, \pi(T) = .03, i(T) = .03$ is achievable from the states below this trajectory with $u \geq -.005$.

We have determined this critical trajectory by running (23)-(25) backwards from $y(T) = 0, \pi(T) = .03, i(T) = .03$ with $u = -.005$. The results are shown in Figure 7 and in the three dimensional Figure 8, which helps to continue the analysis started in Figure 6 upper panel. Also, see Figure 12, page 23 for a general idea on the evolutions of the economy at hand.

![Figure 7: Evolution trajectories at corner C.](image)

The bank controls a “booming” economy to a steady state on the solid line. The instrument (a declining interest rate) is implemented in the period within which the output gap is maximal (4%) and inflation is high but still below 3%. To “cool” the economy down, the interest rate applied has to be close to the limit (which is 7.7%). The interest rate is gradually eased and, after almost 8 quarters, reaches 3%, the same level.
as inflation. This trajectory (solid line) delimits the viability kernel, which is the green area left from the red line. The same trajectory is shown in Figure 8 (and also in Figure 12, page 23).

Figure 7 illustrates also what can happen if the bank does not combat inflation early. Above the solid line are states of the economy, from which violation of the 3% inflation limit will occurs almost surely (in finite time), given the constraints on \( \frac{d\pi}{dt} \).

Consider an output gap of 3%. If inflation is 2.8% (see point E) and the interest rate is e.g. 7.1% the fastest-drop interest rate policy leads the economy outside \( K \) in less than three quarters. The evolution of the economy that corresponds to this policy is represented by the dash-dotted line starting at E. (This evolution is also shown in Figure 8 as the dash-dotted line first from the left). No policy from \( K \) can stabilise the economy if it has reached E. To keep inflation in check, the interest rate would have to be 11% at E, then the economy would move along the thin dashed line originating at E. However, around \( y = 0 \) and \( \pi = 3\% \) the interest rate would be 9.5% hence the real interest rate would be 6.5% and the output gap would soon slip into negative territory.

![Figure 8: 3D evolution trajectories at corner C.](image)

At point F inflation is 2.5%. The dash-dotted line originating at this point shows the economy evolution when the initial interest rate is 7.1% and lowered in the fastest possible manner (\( u = .5\% \) per quarter). After about 6.5 quarters the economy reaches \( y = 0 \) and the real interest rate is about 1%. Hence the contractionary pressure is small at that point. This evolution is also shown in Figure 8; this is the dash-dotted line that starts below (or right to) the thick red line.

If the nominal interest rate at F is close to the upper limit of 7.7% then a fast adjustment interest rate policy leads to \( y = 0 \) in about 5 quarters (see the thin solid line). The corresponding interest rate is 5.1 % and the real interest is 2.3 %. If this policy is continued, inflation drops and output gap diminishes. However, the inflation and interest rates are sufficiently high to avoid a liquidity trap (see Section 6.2.2).
If the initial interest rate at F is low then the fastest-drop interest rate policy may lead to a zero real interest rate before a neutral output gap is realised. With a negative real interest rate and positive output gap, the economy is bound to hit the 3% upper bound on inflation. This evolution is represented by the thin dashed line originating at F. This evolution is also shown in Figure 8; this is the thin dashed line originating below the dash-dotted line.

In general, if the economy is below the critical evolution trajectory, i.e., inside the viability kernel, application of allowable instruments prevents violation of the inflation upper limit.

6.2.2 Liquidity trap

We will now determine the kernel’s boundaries for the “south-west” corner (A). As in the case of corner C, we need to discuss what goal(s) the Reserve Bank may want to achieve when the economy is in “recession” \(y(t) < 0\), when inflation and interest rates are low such that the realisation of a sizeable negative real interest rate \(i(t) - \pi(t)\) is impossible. For \(i(t) - \pi(t) \approx 0\) and \(y(t) < 0\), a long recession is looming, see equations (23)-(24).

However, even if the interest rate is zero a small positive inflation creates a small negative real interest rate, which can stimulate the economy to grow. Of interest to the Bank will thus be an ability to identify the economy states from which the zero interest rate policy guarantees a recovery without sliding to deflation.

In Figure 9 a limiting trajectory has been computed on which the zero interest rate policy is applied i.e., \(i_0 = i(10) = 0\). It is represented by the thick red line that originates at P. The same line is also shown in Figures 10 (and Figure 12); it is marked \(i_0 = 0\). On this line, the economy evolves from \(y(0) < 0\) to \(y(T) \geq 0\) where \(T = 12.9\) quarters and the inflation decreases from about 1.9% to 1%. To calculate this trajectory, we have run (23)-(25) backwards from \(y(T) = 0\), \(\pi(T) = .01\), \(i(T) = .0\) with \(u = 0\) until the output gap’s lower boundary. The trajectory (red solid line) delimits a viability kernel (in green) from where the zero interest rate policy guarantees achievement of a positive output gap in finite time (here, less than 13 quarters). Any state to the left of the solid line does not have that property.

Consider an output gap that is at -3%. If inflation is 1.4% (see point G) then the largest (negative) real interest rate that can be achieved is -1.4%. This is too little to stimulate the economy. The thin solid line, on which this evolution takes place, leaves the constraint set \(K\) with \(y < 0\) in about 4 quarters. We can also see this evolution in Figures 10 (and 12 page 23), where it is represented by the thin solid red line below the separating trajectory.

Consider now point H at which inflation is 1.6% and output gap is -3% as before. Under the zero interest rate policy, the economy reaches \(y = 0\) in about 10 quarters. After that, the inflation increases and output gap grows further. This evolution is represented in Figure 9 by the thin solid (blue) line. The same evolution is also shown in the three dimensions in Figures 10 and 12.

However, the “current” interest rate (i.e., one from which the bank begins its fight against recession) may be ‘high’ and dropping it to zero may create an undesirable “shock” if (25) is not satisfied. To avoid the shock, the Bank needs to change the interest “smoothly”. If so, the negative real interest rate cannot be realised instantaneously and some further shrinking in the output gap will take place. Depending on the initial interest
The rate, the rate at which cuts in the interest rate are “smoothed” and on the strength of the response of the output gap to the interest rate, this secondary ‘shrinkage’ phase in the output gap may encompass and especially lengthy period. In fact, violation of the lower bound on the output gap is imminent if the initial interest rate is high relative to inflation measured at the same time. It is therefore important for the Bank to identify the states of the economy from which the fastest-drop interest rate policy moves the economy to point P (or to the separating trajectory P→0,.01) (see Figure 9) with final interest rate equal to zero. From P or from the separating trajectory, the zero interest rate policy will lead the economy to recovery.

Consider the economy’s evolution for low inflation and small negative output gaps. Figure 11 shows several trajectories that bring the economy to zero interest rate with a maximal interest rate drop. For all such trajectories, the output gap shrinks before it starts growing again.

We can see that for a recovery, which would avoid a large output gap decline, the interest rate $i(t)$ for $y(t) < 0$ should not be too high. We see that maintenance, by the Reserve Bank, of a relationship between the inflation and interest rates is crucial for the fastest recovery.

We can also see that keeping the economy in the viability kernel may prevent a liquidity trap. This could have happened if an economy had dropped outside the viability kernel for a material length of time at a negative output gap level. By remaining in the viability kernel, a deflationary economy can be avoided.

Figure 9, together with equations (23)-(25), can help us understand why the minimum allowable inflation level should be kept positive: if inflation is positive then the interest rate can be made smaller (positive or zero) than the inflation rate and hence a negative real interest rate can be achieved. This helps output gap to grow. Should inflation be non-positive, the output gap will grow more slowly or not at all.
Figure 10: 3D evolution trajectories at corner A.

Figure 11: Corner A: evolutions for maximum-speed recovery.
6.2.3 Evolutions in three dimensions

We infer from Figure 11 that for evolutions that do not escape from $K$, the interest rate has to be lower than some critical level, which depends on the output gap and on inflation. To better understand the viability kernel for corner A we need to examine the system evolutions in $\mathbb{R}^3$.

In Figure 12 we can identify the rectangular box $K$ within which we want system (23)-(25) to evolve. The trajectories shown in this figure are examples of evolutions and were commented on before. Those which remain in $K$ are viable.

![Figure 12: Viability problem presentation in $\mathbb{R}^3$.](image)

In Figure 13, a blow-up of the rectangular box $K$ around corner A. The figure shows the maximum-speed recovery evolutions shown earlier in Figure 11 in $\mathbb{R}^2$. A cone is built by the recovery evolution trajectories: the outermost trajectory is the evolution corresponding to $i_0 = 10.2\%$ shown in Figure 11. The innermost trajectory is corresponds to $i_0 = 0\%$. If the economy is to recover speedily and remain in the rectangular box then the economy coordinates: output gap, inflation and interest rate must remain in the cone intersection with $K$.

In other words, the cone’s interior contains the viable points of $K$, from which a viable evolution is possible. Hence the cone interior is the viability kernel for corner A$^{22}$.

A caution for the conduct of monetary policy is apparent from the above analysis: even mildly negative output gaps can lead to a liquidity trap if inflation is low relative to the interest rate. This is mainly due to inertia of the economic processes considered here. Keeping the process evolution inside a viability kernel guarantees that the instru-

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$^{22}$We can appreciate now that the viability kernels shown in Figures 7 and 9 are $\mathbb{R}^2$ projections of the viability kernel that depends generically on interest rate.
ment (here: nominal short term interest rate) will be applied sufficiently early so that uncontrollable fallout can be avoided.

Figure 13: Corner A blow up.

7 Viable solutions for an economy subjected to shocks

7.1 Time-line for a stochastic economy evolution

The residuals of the processes of output gap and inflation (see Sections 4.2 and 4.3) are measured as the differences between the quarterly observations (historical) and those implied by the fitted model. These differences are the historical “shocks” experienced by the economy each quarter. Hence, the information on the residual distributions gathered in Section 4.2 is also the information on the shock distributions. We assume that future shocks will be drawn from these distributions and propose the following model for the management of the evolution of a stochastic economy:

• at quarter $t$, the data on output gap, inflation and nominal interest rate are known\(^{23}\);
• the decision about an increment of the interest rate for the next quarter is taken;
• the economy develops according to (23)-(25);
• at time $t + h$ where $h = 1$ quarter, the economy is subjected to a two dimensional shock $u(t), \eta(t + 1)$ (see (8), (9)) from the distribution of the residuals (see Section 4.2).

\(^{23}\)Given lags in the publication of economic, for example, inflation data is known only with a one quarter lag, and output data with a two quarter lag. However, it is not unreasonable to assume that a policymaker will have “good” estimates of these variables available in the near term.
We know the deterministic values of output gap and inflation at time $t + 1$, as established by (23)-(25); we will call these values $y(t+1), \pi(t+1)$, respectively. We also know the distributions of “true” states $y(t+1), \pi(t+1)$ at $t + 1$. They are the residuals’ distributions centred at (or shifted to) $y(t+1), \pi(t+1)$.

If we restrict our attention to the shock realisations from intervals $\pm \sigma_y$ and $\pm \sigma_\pi$ where $\sigma_y, \sigma_\pi$ denote estimates of the shocks’ standard deviations then we are able to claim that, at $t + 1$, the economy will “quite likely” be at:

$$\tilde{y} \equiv y(t+1) \pm \sigma_y, \quad \tilde{\pi} \equiv \pi(t+1) \pm \sigma_\pi.$$  \hfill (26)

If the stochastic economy is viable then we should be able to choose a path for the interest rate $i(\tau), \tau \in [t, t+1]$ such that $[\tilde{y}, \tilde{\pi}, \tilde{i}] \in K$ (where $\tilde{i} = i(t+1)$). In other words, a sufficient condition for the viability of the economy is that $\forall t \in \Theta$, there exists an interest rate path such that

$$[\tilde{y}, \tilde{\pi}, \tilde{i}] \in V^K \subset K. \hfill (27)$$

The feasibility of this approach depends on how large $\sigma_y, \sigma_\pi$ are relative to $V^K \subset K$.

In the next section we present an example of a viable policy analysis of the New Zealand economy represented by the simple model considered in the paper.

### 7.2 Monetary policy making in a stochastic environment

Table II presents the observed New Zealand data on inflation and interest rates between 2004:2 and 2004:4, together with ex post estimates of the prevailing multivariate output gap.

<table>
<thead>
<tr>
<th>quarter t</th>
<th>Output Gap $y(t)$</th>
<th>Inflation $\pi(t)$</th>
<th>Interest Rate $i(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004:2</td>
<td>.024213</td>
<td>.02338236</td>
<td>.058567</td>
</tr>
<tr>
<td>2004:3</td>
<td>.021958</td>
<td>.02504603</td>
<td>.064401</td>
</tr>
<tr>
<td>2004:4</td>
<td>.018701</td>
<td>.02749619</td>
<td>.067276</td>
</tr>
</tbody>
</table>


The standard deviations $\sigma_y, \sigma_\pi$ estimated in Section 4.2 are 0.004849 and 0.001497, respectively.

We will show how the viability analysis can help to explain the economy evolution. For example, we will analyse the economy evolution from the beginning of 2004:2 until the end of 2004:4. We will also show the economy state at 2005:1.

Consider the data for the second quarter of 2004. Figure 14 shows where the economy was at this time, see the point marked “∗”; this image is a projection of the 3D representation pictured in Figure 15\textsuperscript{24}. All subsequent “∗” (red) indicate actual outcomes while all “+” (black) represent the model (deterministic) economy.

\textsuperscript{24}Notice that we have allowed for the mean inflation rate, which we have identified to be 2.11%. Therefore the real interest rate which characterises the economy at this quarter, is

$$.058567 - .0211 = 0.037467.$$
The nominal interest rate change between 2004:2 and 2004:3 is 0.58%, which is only slightly above the constraint (22). According to the model (23)-(25), within the next quarter, the “deterministic” economy moves to the point marked by the closest “+”. At this time, the economy is subjected to the two-dimensional shock \((u(2004:3), \eta((2004:3))\). Assuming that the actual shock size will be confined to one standard deviation, we expect the real economy to be in the yellow ellipse, whose long and short axes equal \(\sigma_y, \sigma_\pi\), respectively. The observed historical state of the economy in 2004:3 is represented by the red “∗”.

Figure 14: 2D policy making analysis.

We can see that the real economy is within the one-standard deviation “ball” of the historical shocks.

We can also appreciate that the decision taken in 2004:2 about the interest rate maximum increase was correct insofar as any slower increase in the interest rate would put the economy dangerously close to the viability kernel boundary. As noted in Section 6.2.1, a deterministic economy which crosses the kernel boundary is destined to violate the constraint set \(K\). Obviously, a stochastic economy should keep away from the kernel boundary because of the shocks that might push it outside the kernel.

Now, consider the evolution of the economy in the subsequent quarter i.e., from the end of 2004:3 until the end of 2004:4. The nominal interest rate change between 2004:3 and 2004:4 is 0.29 %, which is below the constraint (22). According to model (23)-(25), within the next quarter, the “deterministic” economy moves to the point marked by the next “+”. At this time, the economy is subjected to the shock \(u(2004:4), \eta((2004:4))\). Again, assuming that the shock size will be confined to the standard deviation range, we expect the real economy to be in the purple ellipse\(^{26}\). The observed historical state of the economy in 2004:4 is represented by the next red “∗”. We can see that the economy is

\(^{25}\)This is almost entirely due to the fact that we are using a 90-day rate as a proxy for the actual policy instrument, the Official Cash Rate. Over this period, even though the OCR itself moved by only 0.5%, the 90-day rate moved by more because several further increases in the OCR were signalled by the Bank.

\(^{26}\)The axes of the ellipse are obviously the same as before. We assume that all shocks are drawn from the same distribution identified in Section 4.2.
within the expected range but very close to the boundary of the viability kernel. Over this period, the central bank signalled a pause in the cycle, believing that monetary settings were appropriate in the current environment in terms of ensuring that the medium term inflation goal of the PTA was met. As mentioned before, this is not captured in our analysis given that the horizon of our model is essentially just one quarter. As judged by the next “∗”, which is well under the separating trajectory, we might conclude that keeping the interest rate change low was justified.

The analysis of the next “+” (black) and its surrounding ellipse might suggest that the shock η(2005 : 1) was negative, probably caused by the NZ dollar appreciation. Also this might have helped the economy to remain within the viability kernel.

8 Concluding remarks

We have considered a simple estimated27 macroeconomic model for analysis based on monetary policy for the Reserve Bank of New Zealand may be established under reasonably general conditions:

I. if \( y(t), \pi(t) \) are well inside \( V^K_F \) apply \( i(t) + \Delta i(t), \Delta i(t) \in [-.005, .005] \) every time interval \( h^{28} \);

II. otherwise apply \( i(t) - .005 \) if \( y < 0 \) or \( i(t) + .005 \) if \( y > 0 \).

These recommendations are in line with policy (7); in particular, (II) is extreme in that it calls for the “full speed” interest rate changes.

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27 We are content to report that our conclusions drawn for the estimated model analysis conform to those reported in [15] where a more abstract analysis of a calibrated model was performed.

28 In our study, \( h \) is a quarter.
We believe there are two states an economy can be in: well inside the viability kernel and close to its boundaries. An assessment of this will obviously depend on the policymaker's judgment. Our graphs are helpful in the assessment. They allow the policymaker to determine where the economy is expected to move to, given current conditions and the applied instruments. If, at \( t + h \), the economy is expected to remain in the kernel then the economy state at \( t \) is well inside the kernel.

The distinction between these two states of the economy is needed for the governor to decide which size of instrument \( \Delta i(t) \) to apply. With our model, the governor can assess where the economy is expected to be at time \( t + h \) and what options he (or she) will then have. We believe that their choices made in this manner will be less arbitrary than the "optimal" ones that rely on the loss function weights and discount rate, which are subjective parameters.

The satisficing policy choices can be modified to allow for measurement errors, parameter uncertainty (like \( \alpha, \zeta \) etc.) and shocks even if the system's dynamics is deterministic. In broad terms, a ball around each point of the trajectory on the \( y, \pi \) plane (see e.g., Figure 6) might be constructed where the ball size is "proportional" to uncertainty. The conditions to apply rules (I), (II) can be modified: if the ball does not intersect with the viability kernel's boundary, apply (I); else apply (II).

In general, the bank's policy established through a viability analysis should appear more credible to economic agents than its optimised counterpart. This might be so because the former depends on fewer arbitrary parameters than does the latter. Notice that this and a few more observations coming below make a viable (or "satisficing") policy less vulnerable to the Lucas critique than its optimising counterpart.

Notice that policy advice based on viability theory may recommend \( \Delta i(t) = 0 \) (see (I)-(II)) for a wide range of the economy states that are well inside the kernel \( V^K_F \). This will mean that the instrument realisations may not change even if \( y, \pi \) have changed. If so, the private sector will have no need to change their parameters hence, the bank's model parameters will not change and the policy will remain time consistent.

In general, a policy based on a viability analysis is precautionary in that it directs the system away from regions of adverse economic conditions (like large negative output gaps or accelerating inflation) where control of the system is difficult or impossible. Hence a viable policy is "naturally" forward looking and attractive under uncertainty. We can say that policy advice based on viability analysis takes a compromise into account between the timing of change in an instrument and its strength.

Future work will concentrate on open economy models, which will take exchange rate uncertainty into account. A starting point for the analysis might be paper [7] that computes viable policies that can keep exchange rate in a target zone; also, see [18] where exchange rate enters the model as a "nuisance" agent's control. Extensions of the model in that the parameters would become new meta-state variables controlled by more "nuisance" agents will produce viable policies robust to private sector agents' reactions hence, even more resistant to the Lucas critique than those computed in this paper.

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29Viability can also deal with explicit stochastic models.
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